Unemployment Insurance Differentiation over the Business Cycle

Andreas Pollak*

Department of Economics, University of Saskatchewan

July 2, 2018

Abstract

This paper quantitatively investigates the welfare implications of varying unemployment insurance (UI) generosity with labour market conditions, in particular over the business cycle. Using a life-cycle model with two-sided matching in the labour market, which is calibrated to match the Canadian economy, I conduct policy experiments that qualitatively confirm results from the theoretical literature that differentiation of UI generosity over the business cycle improves welfare by providing better insurance in times when labour market conditions are difficult. However, I show that quantitatively, the welfare improvements possible though optimal benefit differentiation are very small for UI systems that are reasonably efficient to start with.

Keywords: unemployment insurance, job search, business cycle.

JEL Classification Numbers: E3, J6.

*Address: Arts 922, 9 Campus Dr., Saskatoon, SK, S7N 0P9, Canada, telephone: +1 (306) 966-5221, e-mail: a.pollak@usask.ca. The author is grateful to Mario Centeno, Mark Strøm Kristoffersen, Christian Scharrer, seminar and workshop participants at ZEW, CBS, Freiburg and IFO Dresden and participants of the meetings of the CEA 2011 and 2013, AIEL 2013 and SAEe 2013 as well as two anonymous referees for helpful comments and suggestions. This research has been enabled by the use of the excellent computing resources provided by WestGrid and Compute/Calcul Canada. The research and analysis are based on data from Statistics Canada. The opinions expressed do not represent the views of Statistics Canada.
1 Introduction

The Canadian Employment Insurance (EI) program is unusual in its design compared to other unemployment insurance (UI) programs, as it includes mechanisms to make benefits more easily available when the unemployment rate is high. Specifically, benefit durations depend on the number of hours worked during a qualifying period. In regions where the unemployment rate has been high in recent months, fewer hours are required to qualify for a given benefit duration and the maximum durations are longer.\(^1\) While these features of the system have traditionally been seen as a means of interprovincial redistribution, they do have implications for the effectiveness and efficiency of consumption insurance provision over business cycle if the unemployment rate varies enough over time. This dimension of benefit differentiation, where UI is adjusted over time with changing macroeconomic conditions, has recently been a focus of the literature on unemployment and job search. Moreover, policymakers have experimented with it following the most recent recession.\(^2\)

This paper calibrates a model to Canadian labour market conditions in order to quantify the welfare effects of UI differentiation over the business cycle. Our objective is to investigate the scope for welfare improvements from business-cycle dependent UI generosity in general and provide an assessment of the properties of UI systems regarding consumption insurance against business cycle effects.\(^3\)

The quantitative policy experiments presented in this paper show that while it may be desirable in principle to respond to changes in labour market conditions by adjusting the

---

\(^1\)See appendix A for details.

\(^2\)The UI system in the US also employs a mechanism that can lead to automatic increases in benefit durations during recessions. “Extended Benefits,” which increase the benefit duration by 13 or 20 weeks beyond the usual 26 weeks, are triggered by a combination of high unemployment rates and strong increases in unemployment in a state.

Another country where benefit durations depend on the regional unemployment rate is Poland. However, because extended durations are triggered by high unemployment relative to the national average and only with long lags, the Polish system is unlikely to provide particularly effective insurance against business cycle effects. See Sienkiewicz (2011).

\(^3\)I will use the term generosity to broadly refer to the many aspects of UI that determine the value of the system to unemployed agents, including benefit levels or replacement ratios, maximum benefit durations, any applicable waiting periods, as well as qualification periods and criteria. In later sections, a more formal and narrow definition of generosity will occasionally be used.
generosity of UI systems, quantitatively the possible welfare improvements resulting from fine-tuning the system along this dimension are very small.

Generally, the key to designing an efficient UI system is to find the right trade-off between the consumption insurance that a generous system provides and the moral hazard it induces. A wide range of policies that let benefits follow time-dependent profiles have been studied.\(^4\) Tatsiramos and van Ours (2012) provide an up-to-date survey.

The basic idea underlying the time-varying benefits that are the focus of this paper is that during recessions, unemployment is high but the labour market is slow. This means that on the one hand, making benefits more easily available can substantially improve consumption insurance, while at the same time the effect of such a measure on overall unemployment is likely to be lower than during a boom, as the chances of re-employment are low anyway, leaving less scope for moral hazard.

This type of time-varying UI scheme has been proposed by Kiley (2003) and Sánchez (2008) in a repeated moral hazard framework similar to Hopenhayn and Nicolini (1997). Andersen and Svarer (2010) use a Mortensen-Pissarides search model to show that an optimal system would be substantially more generous than the US system during recessions. Kroft and Notowidigdo (2010) find similar results in a model with stochastic wage offers similar to Shimer and Werning (2007). Mitman and Rabinowich (2011) emphasize that optimal benefits may follow more complex paths over the business cycle. They argue that UI systems should become less generous over the course of a recession to speed up recovery and be procyclical overall. Another possible reason for procyclical benefits may be the requirement to balance the government budget every period as discussed in Andersen and Svarer (2011). Determining whether optimal benefits generosity is procyclical or countercyclical has been one focus of this literature. The key contribution of Landais et al. (2010) is to show how the optimality of benefits can be assessed in any state using an extended Baily-Chetty criterion.\(^5\)

\(^4\)The aspect that has received the most attentions is how benefits should depend on the duration of the unemployment spell. See Hopenhayn and Nicolini (1997) and the literature that followed.

This criterion characterizes optimal UI generosity in terms of consumption risk, risk aversion and moral hazard costs. More specifically, it can be shown that an efficient system exposes households to consumption risk such that the relative increase of marginal utility during unemployment equals the elasticity of unemployment duration with respect to benefits.\footnote{See appendix B for a simple application of the criterion to the problem of optimal benefit differentiation.} Landais et al. extend the Baily-Chetty formula to account for the general-equilibrium search externality present in two-sided search models.\footnote{Other contributions to this literature include Ek (2012), Jung and Kuester (2011), Moyen and Stähler (2009) and Schuster (2012).}

Due to the complexity of the policy problem at hand, theoretical models of time-varying unemployment insurance regularly incorporate strong assumptions that may limit applicability of the results in a real-world context. In particular, all the papers mentioned above assume hand-to-mouth consumption and therefore rule out the possibility that households self-insure through saving.\footnote{It should be noted, however, that the criterion developed in Landais et al. likely generalizes to the case of endogenous savings, just as Baily’s criterion does.} It is one of the contributions of this paper to examine business-cycle dependent UI in a more realistic general-equilibrium setting with fully optimizing agents. Another paper that does this is Kristoffersen (2012), who investigates optimal UI systems in a general equilibrium model similar to Krusell et al. (2010) with fully optimizing, infinitely lived households. The author finds that, in line with the arguments of Landais et al. (2010), either procyclical or countercyclical benefits can be optimal, but that the welfare consequences of allowing for differentiated UI generosity over the business cycle are extremely small.\footnote{In contrast to our model, Kristoffersen (2012) employs an infinite horizon setting rather than a life-cycle model. Moreover, he does not allow for moral hazard of any kind.}

There is a growing literature that investigates the effects of extensions to benefit durations during recessions empirically. Farber and Valetta (2013) and Rothstein (2011) use data from the American Current Population Survey (CPS) to show that extended benefit durations lead to small but significant extensions in unemployment durations. Schmieder et al. (2012) find similar results in German data.\footnote{Even though benefit durations do not vary with labour market conditions in Germany, the authors are...} Using a different approach based on a simulated structural
model with fully optimizing households whose productivity depends on their employment history, Nakajima (2012) attributes almost a third of the rise in US unemployment during the 2009-2011 period to extended benefit durations.

The plan for this paper is as follows. Sections 2 and 3 describe the model and discuss its calibration and simulation. The main results are presented in section 4. Section 5 includes a number of robustness checks and highlights the importance of labour market outcomes of young households for the overall welfare effects. The final section of the paper concludes.

2 The Model

In any given period $t$, the economy is in one of two possible macro states $s_t \in \{0, 1\}$. In the bad state $s = 0$, which will be identified with periods of recession, total factor productivity (TFP) $p(s_t)$ is lower and the risk of job loss is higher. The sequence of macro states follows a Markov process.

We model a small open economy, where capital is supplied elastically by the world capital market at the interest rate $r$.

2.1 Households

The economy is populated by equally large overlapping generations of heterogeneous agents. Each individual has a life span of 60 years, corresponding to ages 20 to 79. Important events include entry into the economy and the labour market at age 20, retirement at age 65 and death at 80 years of age. All agents are identical initially; however, their actual income profiles over their lives differ due to the idiosyncratic nature of labour market opportunities, resulting in different asset levels and consumption possibilities among individuals of the same cohort.

When they first enter the economy, agents are endowed with assets corresponding to three
months of labour income\textsuperscript{11}, but they do not have a job. If an individual is unemployed in period $t$, she receives job offers at the rate $\omega(s_t)$. These employment opportunities differ in productivity and are therefore characterized by different wages.\textsuperscript{12} Once an agent receives an acceptable offer, a job is created and production begins in the following period. To the extent that individual reservation wages (or equivalently reservation match qualities) deviate from the social optimum, an inefficiency arises.\textsuperscript{13} Jobs last until they are destroyed, which happens at the exogenous state-dependent rate $\lambda(s_t)$, or when the agent reaches her retirement age. After retirement, all agents must rely on assets and interest income for consumption.

In every period of their lives, agents maximize the expected utility derived from consumption during the rest of their deterministic lifetime, which is assumed to be time separable.

$$\max E_t \left[ \sum_{s=t}^{T_i} \beta^{s-t} u(c_{is}) \right]$$

Here, the index $i$ refers to the individual, $T_i$ is the last period of agent $i$’s life, which depends on their age\textsuperscript{14}, $\beta > 0$ is a discount factor and $u(c_{is})$ is the agent’s instantaneous utility as function of consumption $c_{is}$.

Agents can save at the prevailing interest rate $r$, but they are unable to borrow.\textsuperscript{15} For every period $t$ of an agent’s life, her financial wealth $a_{it}$ therefore evolves according to the budget and borrowing constraint

$$a_{it+1} = (1 + r)(a_{it} + y_{it} - c_{it}) \geq 0,$$  \hspace{1cm} (BC)

\textsuperscript{11}Three months of the income earned by a 20-year-old agent in a job with match quality one and a reservation wage of zero.

\textsuperscript{12}This mechanism is similar to the one used by Shimer and Werning (2007) and Kroft and Notowidigdo (2010).

\textsuperscript{13}This can be interpreted as agents not accepting jobs that the employment agency would like them to take, i.e. a moral hazard problem that exists because agents who decline jobs are not sanctioned by the government. Pallage and Zimmermann (2005) argue that such sanctions are an important mechanism to control moral hazard in reality.

\textsuperscript{14}All agents have identical life spans.

\textsuperscript{15}I assume that the interest rate is exogenous, which is probably a reasonable approximation for a small open economy such as Canada. This assumption also simplifies the model considerably compared to a closed-economy model with time-varying interest rates as in Krusell and Smith (1998).
where \( y_{it} \) is any income received during this period, which could be wages or UI benefits.

The Bellman equation characterizing the problem of an unemployed household of age \( g \) in period \( t \) that has at least one more period in the labour market left is given by

\[
V^u_g(a_{it}, b_{it}, d_{it}, s_t) = \max_{a_{it+1}, m} \left( u(a_{it} + y^u(b_{it}, d_{it}) - \frac{a_{it+1}}{1 + r}) + \right.
\]

\[
\beta F_{s_t}(m) E_t[V^u_{g+1}(a_{it+1}, b_{it}, d_{it+1}, s_{t+1})] + \beta \int_{-\infty}^{\infty} E_t[V^e_{g+1}(a_{it+1}, w(m, m), s_{t+1})] dF_{s_t}(m) \right). \tag{2}
\]

Here \( b_{it} \) is the individual’s benefit amount or last wage, which is determined upon job loss.\(^{16}\) \( d_{it} \) encodes the remaining benefit duration.\(^{17}\) Given these two variables, it is possible to determine the actual benefit payment \( y^u(b, d) \). The reservation match quality an agent chooses given her current situation is \( m \). \( F_{s_t}(m) \) is the distribution function of the best match quality offered per period in state \( s_t \).\(^{18}\)

If the agent receives a job offer with an acceptable match quality \( m \geq \underline{m} \), Nash bargaining takes place over the share of the output that is paid as a wage. The variable \( w \in [\underline{m}, \bar{m}] \) reflects the outcome of these negotiations and determines the labour income \( y^e \) the household will earn for the duration of the job.

The maximization problem of an employed agent of age \( g \) who is not yet about to retire is

\[
V^e_g(a_{it}, w_{it}, s_t) = \max_{a_{it+1}} \left( u(a_{it} + y^e(w_{it}, s_t, g) - \frac{a_{it+1}}{1 + r}) + \right.
\]

\[
\beta(1 - \lambda(s_t)) E_t[V^e_{g+1}(a_{it+1}, w_{it}, s_{t+1})] + \beta \lambda(s_t) E_t[V^u_{g+1}(a_{it+1}, b(w_{it}, s_t, g), d(s_t), s_{t+1})] \right). \tag{3}
\]

The household’s labour income \( y^e \) may vary over time, reflecting changes in its age-dependent

---

\(^{16}\)\( b_{it} = 0 \) for agents who have never had a job and are therefore not entitled to receive benefits.

\(^{17}\)This variable may, however, also be used to determine whether the agent is in his waiting period.

\(^{18}\)\( m = 0 \) corresponds to the case of zero offers in that period.
productivity and the aggregate state. If the job is destroyed at the end of the period, the agent’s entitlement to UI benefits is determined as a function of her current income and the macro state, $b_{st+1} = b(w_{st}, s_t, g)$ and $d_{st} = d(s_t)$.

A retired individual’s problem is simple and deterministic.

$$V_g(a_{st}) = \max_{a_{st+1}} \left( u(a_{st} - \frac{a_{st+1}}{1+r}) + \beta V_{g+1}(a_{st+1}) \right),$$

where $V_g(a_{st}) \equiv 0$ if the age $g$ exceeds the agent’s life span.

### 2.2 Firms

The labour market is characterized by a Mortensen-Pissarides setting with two-sided search. In order to produce output, firms must first post a vacancy, then wait for a worker to be matched and negotiate a contract. Unfilled vacancies are associated with a flow cost of $kp(s_t)$ that is proportional to TFP. The frequency of matches between workers and firms is determined by an aggregate matching function. Following the literature on job search, I assume that this function is Cobb-Douglas with constant returns to scale, so that the rate of job offers unemployed workers receive can be written as $\omega(s_t) = \bar{\omega}(\theta(s_t))^\eta$, where $\theta(s_t)$ is the market tightness as a function of the macro state.

Each firm (or job) produces output for a competitive goods market using capital and labour as inputs. The production technology is Cobb-Douglas with a capital share of $\alpha$. Capital depreciates at the rate $\delta$. The productivity of a worker $i$ on a job $j$ depends on two factors, the worker’s inherent productivity $q_{it}$ and the quality of the match $m_{ij}$. $q_{it}$ increases deterministically over an agent’s working life, whereas $m_{ij}$ is a measure of how good the applicant fits the job description. For each match, this match quality $m_{ij}$ is drawn from a lognormal distribution, $\ln m_{ij} \sim N(-\frac{1}{2}\sigma_m^2, \sigma_m^2)$. $m_{ij}$ and $q_{it}$ together determine the effective labour supplied by worker $i$ on job $j$, so that the output produced is given by $Y_{ijt} = p(s_t) (m_{ij} q_{it})^{1-\alpha} K_{jt}^{\alpha}$. The capital input $K_{jt}$ is adjusted optimally every period,

---

19See Pissarides (2000).
implying $Y_{ijt} = \left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}} p(s_t)^{\frac{1}{1-\alpha}} m_{ij} q_{it}$.

2.3 The Labour Market and Bargaining

When a firm and a worker meet, Nash bargaining over the part of the labour share $(1-\alpha)Y_{ijt}$ that is paid to the worker as a wage $y_{ijt}^e$ takes place. Once a job is created, the ratio $\frac{y_{ijt}^e}{Y_{ijt}}$ stays constant as wages are adjusted to reflect changes in aggregate and individual productivities.\textsuperscript{20}

Specifically, let

$$J_{it}(m) = E_t \sum_{\tau=t}^{\infty} (1 + r)^{-(\tau-t)} y_{ijt}^e I_{i\tau}$$

$$= E_t \sum_{\tau=t}^{\infty} (1 + r)^{-(\tau-t)} (1 - \alpha) \left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}} p(s_t)^{\frac{1}{1-\alpha}} m_{ij} q_{it} I_{i\tau}$$

be the expected present value of the labour share of output associated with a job created in period $t$ with worker $i$ as a function of the match quality $m$. $I_{i\tau} = 1$ if the match with agent $i$ still persists in period $t$, zero otherwise. The negotiated wage parameter $w$ determines the payment to the worker in every period, $y_{ijt}^e (1-\alpha) \left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}} p(s_t)^{\frac{1}{1-\alpha}} w q_{it}$, so that the worker’s expected present value of income from this job is $J_{it}(w)$. $w$ maximizes the generalized Nash product

$$N(w) = (J_{it}(m) - J_{it}(w))^{1-\epsilon} (J_{it}(w) - J_{it}(w))^{\epsilon},$$

where $m$ is the actual match quality of the job, $w = \underline{m}$ is the worker’s reservation wage (or reservation match quality)\textsuperscript{21} and $\epsilon$ is the agent’s bargaining power. Since $J_{it}(m)$ is proportional to $m$, $J_{it}(m) = m J_{it}(1)$, the result of the bargaining process is given by $w = w(m, \underline{m}) = w(m, m) = \underline{m} + \epsilon(m - \underline{m})$. This wage parameter could be interpreted as the job’s effective match quality from the agent’s perspective, a measure that accounts for the fact

\textsuperscript{20}This \textit{ad hoc} assumption implies that while wages do respond to productivity changes, they do not change over time in response to labour market conditions.

\textsuperscript{21}A job with the match quality $\underline{m}$ generates no quasi-rent, and thus the full amount of the labour share is assigned to the worker.
that part of the labour share will go to the firm.

The market tightness in each state is determined by the condition that the expected value of a vacancy be zero. In other words, the aggregate vacancy costs in each state are equal to the rents earned by firms on jobs created in the same state.\textsuperscript{22}

\section*{2.4 The Government}

The government runs a UI system. It makes payments to eligible working-age individuals. Moreover, the government provides new agents with their initial asset allocation.\textsuperscript{23} To pay for these expenditures, the government levies a proportional tax at a constant rate on all labour income and transfers, i.e. on wages, UI benefits and the agents’ initial endowment.\textsuperscript{24}

\section*{2.5 Equilibrium and Welfare}

We will not consider transitions paths. The focus will be on long-term equilibria, in which the cross-sectional distribution of the population with respect to individual characteristics is stationary.

The welfare criterion used is the cross-section average of the utilities of all agents who have successfully entered the labour market.\textsuperscript{25} Whenever welfare effects of policy changes are

\textsuperscript{22}I make the simplifying assumption that the market tightness depends on the current state only. This neglects the possibility of using the current distribution of household characteristics to predict time-varying reservation wages more accurately. This simplification is necessary for computational reasons, and is unlikely to be quantitatively important.

\textsuperscript{23}Note that this means that the tax rate will be positive even if no UI benefits are paid.

\textsuperscript{24}The assumption that all wages and transfers are taxed at the same rate makes it possible to solve the model independent of tax policy in the case of CRRA preferences.

\textsuperscript{25}Agents who have not yet successfully entered the labour market are excluded for simple practical reasons. As these individuals are all unemployed but not eligible to receive UI benefits, their welfare tends to depend strongly on the prevailing market tightness. Generally, these agents favour less generous systems as they imply a higher matching rate and lower taxes on initial wealth. Depending on the assumptions regarding young agents’ initial endowment with assets, the impact of the welfare of this group on overall welfare may be anywhere from negligible to dominating. In the absence of a serious calibration of the average level and cross-sectional variation of initial financial wealth or access to other means of support - such as social assistance or support from relatives - I decided to remove this group entirely from the welfare calculation. This allows us to focus on the business-cycle properties of UI rather than distributional effects regarding young unemployed.

Note that this issue does not arise in an infinite-horizon setting where everybody has an employment history and is thus potentially eligible to receive UI benefits.
reported, they are translated into equivalent relative changes in consumption, i.e. a reported 1% welfare improvement is equivalent to the welfare change that would have resulted from a general 1% increase in consumption.

3 Calibration and Computation

The model is calibrated to match the aggregate Canadian economy. Whenever possible, the model economy replicates features of the Canadian economy between July 2000 and December 2012. This time frame is chosen to make use of all available microdata on EI economic regions in their current configuration, which has been in place since July 2000. These 12\(\frac{1}{2}\) years are a short time for measuring business cycle effects, however. This period only includes one relatively short recession. For this reason, some parameters are calibrated using a longer data period from January 1976 to June 2013.\(^{26}\) The starting point of this extended period was chosen to match the introduction of the Labour Force Survey (LFS), the main source of labour market statistics in Canada, in its present form.

For the calibration exercise, I use a specification of the unemployment insurance system that resembles the Canadian Employment Insurance. When conducting the policy simulations presented in the following section, this system will be replaced by alternative UI specifications, while all other parameters characterizing the economy will remain the same. The system is modelled as follows. Benefits are paid after a waiting period of two weeks at a replacement ratio of 55\%.\(^{27}\) In practice, the duration of the benefits depends on the prevailing unemployment rate as well as an individual’s number of hours of insurable work during a qualifying period. Since the work requirement is relatively low compared to the typical time between two unemployment spells, I assume that everyone is entitled to receive benefits for the maximum duration available to those with about one year of full-time emp-

\(^{26}\) This extended period is three times as long as the shorter period – 37\(\frac{1}{2}\) years or 450 months vs. 12\(\frac{1}{2}\) years or 150 months. Recessions are occurring more frequently in the extended period, accounting for 10\% of all quarters compared to 6\% in the shorter period.

\(^{27}\) There is no ceiling to contributions or EI benefits in the model.
ployment. Benefit durations depend on whether the unemployment spell begins during a boom or a recession. For the aggregate economy, average unemployment rates of 7% to 8% in the good state and 9% to 10% during recessions imply benefit durations of 40 and 44 weeks, respectively.

The households’ instantaneous utility function $u$ is assumed to exhibit a constant relative risk aversion of $\gamma = 3$. The annual discount rate is 4%. These values are standard in the macroeconomic literature.

The interest rate is set to the average long-term interest rate of 2.4% for the 2000:7 to 2012:12 period.\footnote{The real rate of return on 10-year government bonds.}

Individual productivity $q_{it}$ increases deterministically in a piecewise linear fashion with age to generate lifetime income profiles that resemble those found in household panel data.\footnote{See e.g. Rupert and Zanella (2010).} It rises by 60% between ages 20 and 35, and then by another 20 percentage points until retirement.

The transition probabilities between the macro states are chosen to match the average frequency and duration of recessions in Canada between 1976:Q1 and 2013:Q2. The average duration of recessions during this time was $3\frac{3}{4}$ quarters, with recessions accounting for exactly 10% of the periods.

Productivity is set to be 5.1% lower during recessions, which is the average multifactor productivity (MFP) differential for the business sector between 1976 and 2009 as reported by Statistics Canada.\footnote{Multifactor productivity is reported at annual frequency, with 2009 being the most recent year available.} Production is characterized by a capital share of $\alpha = \frac{1}{3}$ and a depreciation rate of $\delta = 8\%$ per year.

The matching parameter $\eta$ is set to $\frac{1}{2}$, in line with the empirical literature.\footnote{See for example the comprehensive survey of Petrongolo and Pissarides (2001) or the more recent literature cited in Barnichon and Figura (2015).} The variance of match qualities of $\sigma_m^2 = 1\%$ is consistent with the estimates reported in Pollak (2013).

The remaining parameters are calibrated such that the simulated model matches certain
aspects of the data. Specifically, for the calibration of the aggregate economy, the matching rate \( \omega(1) = 0.323 \) per period is chosen to generate an unemployment rate of 7.13% in the good state, equal to the 2000:7-2012:12 average. Market tightness in the high state is normalized to \( \theta(1) = 1 \). The matching rate in the low state is then implied to be \( \omega(0) = 0.306 \), given the lower market tightness \( \theta(0) = 0.898 \) resulting from lower profits during recessions. The job destruction rate during recessions of \( \lambda(0) = 1.43\% \) per period implies an average monthly increase of the unemployment rate of 0.25 percentage points, matching the respective value for the Canadian economy between 1976:1 and 2013:6. Finally, \( \lambda(1) \) is calibrated to 0.90% in order to match the average unemployment duration of 13.8 weeks found in the LFS for the 2000:7 to 2012:12 period.

The household’s relative bargaining strength is set to 95%. This value implies a vacancy cost of \( k = 0.58 \) in the model calibrated to the aggregate economy. In the simulations reported below, market tightness in the good state is normalized to one. In this case, the vacancy cost is about 5.8% of the average wage. Adjusted to the market tightness of about \( \frac{1}{6} \) as reported by Statistics Canada, the vacancy-cost-to-wage ratio of about one third is consistent with the values reported in the empirical literature.

The calibrated parameters for the aggregate scenario are summarized in table 1.

The model is simulated at a period length of \( \frac{1}{2} \) month, i.e. at 24 periods per year or 1440 periods in a lifetime. To solve the model for a particular UI system and parameter constellation, it is necessary to determine the market tightness values. This is done as follows. First, the household’s problem is solved for a given set of tightness values, \( \theta(0) \) and

\[ \text{Note, however, that in our model agents are faced with a wide range of job offers in terms of match quality. The high bargaining strength of the household merely reflects the idea that workers receive most of the rent from picking a job that fits their qualification rather than a random job they are less qualified for.} \]

\[ \text{See series v65958686 and v65958994 for the number of vacancies and unemployed at monthly frequency. Reliable vacancy data for Canada has only been available since 2011. Please note that the job vacancy rates reported by Statistics Canada are based on a narrow definition of unemployment and thus differ from the market tightness concept used in the theoretical literature.} \]

\[ \text{This is essentially the same vacancy cost that has been used by Landais et al. (2010). Empirical studies tend to find values between about 10\% and 60\%, see Barron et al. (1997), Silva and Toledo (2005) and Hagedorn and Manovskii (2008).} \]

\[ \text{Further details regarding the data used in the calibration can be found in appendix C.} \]
Table 1: Calibration: Aggregate Canadian Economy

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>matching parameter to data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>3.0</td>
<td>relative risk aversion</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.96$^a$</td>
<td>discount rate</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$\frac{1}{3}$</td>
<td>capital share</td>
</tr>
<tr>
<td>$\delta$</td>
<td>8%$^a$</td>
<td>depreciation rate</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.95</td>
<td>household’s bargaining power</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$\frac{1}{2}$</td>
<td>matching parameter</td>
</tr>
<tr>
<td>$\sigma^2_m$</td>
<td>0.01</td>
<td>variance of match quality</td>
</tr>
<tr>
<td>$r$</td>
<td>2.41%$^a$</td>
<td>10-year avg. yield of government bonds</td>
</tr>
<tr>
<td>$p(0)/q(1)$</td>
<td>0.949</td>
<td>drop in MFP during recessions</td>
</tr>
<tr>
<td>$s = 0 \rightarrow s = 0$</td>
<td>0.956$^b$</td>
<td>avg. duration of recessions</td>
</tr>
<tr>
<td>$s = 1 \rightarrow s = 1$</td>
<td>0.995$^b$</td>
<td>avg. time between recessions</td>
</tr>
</tbody>
</table>

$^a$ per year

$^b$ per period ($\frac{1}{24}$ year)

$\theta(1)$. Then the economy is simulated for a large number of periods. The market tightness values are adjusted based on the simulation results, with the objective to make expected profits in the firm sector zero. This process is repeated until convergence.$^{36}$

The household’s problem is solved by recursively finding the value function for each of the 1,440 periods of an agent’s lifetime, starting with the last one. The functions are calculated on a multidimensional grid, using linear interpolation between grid points.$^{37,38}$

$^{36}$ A total of 1,440,000 periods (60,000 years) is simulated with a cohort size of one person and 1,440 overlapping cohorts at any point in time. The convergence criterion used is a deviation of the market tightness from its predicted value by less than 0.1%.

$^{37}$ Further details on the numerical solution strategy are provided in appendix D.

$^{38}$ Solving the periods prior to retirement is computationally much harder because of the larger number of
4 Results

This section investigates how the business cycle differentiation of replacement ratios and benefit durations impacts welfare. This is done by using the calibrated model of the aggregate Canadian economy to perform a number of simulations under different UI systems and comparing characteristics of the resulting equilibria.\textsuperscript{39} All the UI systems considered in this exercise have the following properties: Agents who lose their job are entitled to receive benefits. The benefit duration is determined at the time of job loss, like in the Canadian EI system. This duration may be different depending on whether the spell begins during a good or bad state. There is no waiting period. The replacement ratio may also depend on the state of the economy, but at any point in time all agents are subject to the same replacement ratio, i.e. the replacement ratio for an individual may change with the state of the economy. The UI system is therefore characterized by four parameters, two replacement ratios $\rho(s)$ and two benefit durations $d(s)$.\textsuperscript{40}

It is important to note that any such system, where benefits are provided at a duration-independent rate for a given time, is unlikely to be a good approximation to the optimal UI system, which efficiently balances re-employment incentives with insurance considerations. Such a system would typically combine a one-time payment, compensating households for the expected earnings loss, with a time-varying UI benefit that protects workers against the low risk of very long unemployment durations. Actual UI systems typically pay relatively high replacement ratios for a limited time, thus distorting job search behaviour for the majority of jobseekers while providing little protection against longer unemployment spells.

As we are interested in the effects of differentiating a UI system over the business cycle,
Figure 1: Optimal differentiation of benefits over the business cycle

Note: Upper panels: Optimal policy parameter in recessions as a function of policy parameter during non-recessions (solid line) compared to no differentiation over the business cycle (dashed line). The shaded area shows the range of policy parameters that yield welfare within \( \frac{1}{100} \) % of the optimal choice. Lower panels: Relative welfare improvement from choosing optimal differentiation over no differentiation as a function of the policy parameter during non-recessions. Left: Varying replacement ratio for a fixed benefit duration of 3 months. Right: Varying benefit duration for a fixed replacement ratio of 30%. Values obtained for a 4th-order polynomial approximation of simulated welfare function.

looking at how the differentiation of replacement ratios and benefit durations affects welfare is a good starting point. The solid line in the top left panel of Figure 1 shows the welfare-maximizing choice of replacement ratios during recessions, \( \rho(0) \), as a function of replacement ratios in normal times, \( \rho(1) \) for a constant benefit duration \( d(s) = 3 \). The dashed line corresponds to undifferentiated replacement ratios, \( \rho(0) = \rho(1) \). The optimal level of \( \rho(0) \) rises monotonically with \( \rho(1) \) and is above \( \rho(1) \) for replacement ratios below 40%. In this range, countercyclical UI generosity is desirable. For \( \rho(1) > 40\% \) generosity should be procyclical. As pointed out by Landais et al. (2010), the two factors determining the optimal benefit generosity at the margin are its effect on the insurance-cost trade-off as summarized by the Baily-Chetty condition and its general-equilibrium effect on the search externality. Given our calibration, the second effect always works towards countercyclical generosity. The
optimal undifferentiated policy is $\rho(0) = \rho(1) = 24\%$ while the optimal differentiated policy is $\rho(0) = 34\%$ and $\rho(1) = 23\%$. Clearly, there is a benefit to a differentiated policy that outweighs its costs well beyond the optimal level of generosity. For $\rho(1) > 40\%$, however, the high of procyclical replacement ratios that result from longer unemployment durations make them unattractive from a welfare perspective.

Figure 2: Market Tightness and Replacement Ratios

![Figure 2: Market Tightness and Replacement Ratios](image)

*Note:* Market tightness in the bad state (solid line) and on average (dashed line) as a function of the replacement ratio in the bad state, assuming $\rho(1) = 0.5$ and a benefit duration of 3 months.

Even though the optimal level of $\rho(0)$ may differ substantially from $\rho(1)$, the welfare cost of choosing an undifferentiated system over the policy involving the optimal replacement ratio during recessions is very low. The gray area in the figure shows the range of $\rho(0)$ that involves a welfare reduction of less than $\frac{1}{100}\%$ compared to the optimal choice of $\rho(0)$. Note that the dashed line representing $\rho(0) = \rho(1)$ lies within this range for $\rho(1) < 90\%$, which means that the cost of the undifferentiated system compared to the optimally differentiated system is small. The lower left panel of figure 1 explicitly shows this welfare cost.

The fact that the welfare effects of picking the “wrong” replacement ratio during recessions is small does not mean that this policy parameter has no effect. Figure 2 shows how
the market tightness in the bad state drops with the corresponding replacement ratio. However, even though labour market conditions change fairly strongly, this only has a significant impact on a relatively small number of periods (recessions) on a relatively small number of households (those who are unemployed then), and only affects them in terms of welfare to the extent that they are not fully compensated for changes in job finding opportunities with changes in benefits.

The two panels on the right of figure 1 show the corresponding effects of varying benefit durations instead of replacement ratios. For a fixed replacement ratio of \( \rho(0) = \rho(1) = 30\% \) the optimal benefit duration \( d(0) \) is shown as a function of \( d(1) \) in the upper panel. In this scenario, countercyclical generosity is desirable for benefit durations of up to \( 2\frac{1}{2} \) months. An interesting feature of the optimal benefit duration is that it is not a continuous function of \( d(1) \). The discontinuity results from the fact that the welfare function is generally non-concave in \( d(0) \).\(^{41}\) Again, the welfare costs of choosing an undifferentiated policy \( d(0) = d(1) \) over the optimal differentiated one for any level of \( d(1) \) is very small.\(^{42}\)

In both of the above scenarios that vary either replacement ratios or benefit durations taking the other policy variable as given, it may be optimal to implement a considerable differentiation of UI over the business cycle. Under our parametrization, benefit generosity should be procyclical in the vicinity of the optimal non-recession parameter value.

It is tempting to search for the welfare-maximizing combination of the UI parameters \( \rho(s) \) and \( d(s) \). However, allowing both replacement ratios and benefit durations to vary at the same time yields one of two extreme, albeit perfectly sensible, solutions to the optimization problem.

One possibility is that it is optimal to choose a very low replacement ratio to reduce the adverse effects on the rate of job acceptance. Once the moral hazard problem is dealt with in this way, however, there is no need to restrict the benefit duration any more. As

\(^{41}\)Note that the discontinuity occurs for a rather high value of \( d(1) \), more than five times the optimum.  
\(^{42}\)In this scenario, the optimal undifferentiated policy is \( d(0) = d(1) = 2.0 \) months and the optimal differentiated policy is \( d(0) = 2.4 \) months and \( d(1) = 1.9 \) months.
a consequence, a system under which households can receive a very low benefit indefinitely may be optimal.\textsuperscript{43}

The other possibility is given by the opposite limiting case along the duration dimension. As the negative incentive effects of effectively paying agents for being unemployed can be reduced for any given level of generosity by reducing the benefit duration and increasing the replacement ratio accordingly, it may be optimal to choose a fixed one-time payment at the time of separation.\textsuperscript{44} In the case of our discrete-time model, the best benefit duration would therefore be one period. In this scenario, moral hazard effects are so strong that agents are willing to give up insurance against unexpectedly long unemployment durations in favour of a mechanism that merely compensates them for the \textit{expected} cost of job loss.

Figure 3 shows the relationship between benefit duration and welfare for a given level of maximum payments per spell or overall “generosity” $\Gamma(s) = \rho(s)d(s)$ and a system that is not differentiated over the business cycle. Starting at the lowest possible benefit duration $d(0) = d(1) = 1$ period $= \frac{1}{2}$ month, benefit duration is expanded up to a maximum of 18 months while the replacement ratios are reduced proportionally. The resulting U-shaped relationship clearly shows that the most efficient systems are associated with either very short benefit durations or very low replacement ratios. These are the systems that provide the best approximations to the optimal UI systems. Systems with intermediate benefit durations are less efficient, even though the resulting welfare reductions are quite small.

Table 2 compares Canadian EI to the best UI systems in two alternative classes. For each class, the best systems with and without benefit differentiation over the business cycles are presented.

The first class of UI systems does not limit benefit duration, $d(s) = \infty$. It contains the

\textsuperscript{43}Pollak (2007) argues that it would be preferable to provide these benefits in the form of fixed social assistance payments that are independent of previous earnings.

\textsuperscript{44}Another effect that works towards making a one-time payment preferable is a discounting or interest effect. Everything else equal, households prefer to get payments earlier. To the government it does not matter whether it pays households today or tomorrow, because in either case it is just redistributing from one group to another. This effect arises in part because we are comparing steady states without taking into account the one-time expenditure or revenue resulting from the introduction of the system.
Figure 3: Welfare as a Function of Benefit Duration (in Months) for Constant Generosity

---

**Note:** Generosity $\Gamma(s) = \rho(s)d(s) = 0.8$ for $s \in \{0,1\}$. Welfare normalized to 1 for the minimum duration of $d(s) = 1/2$ month.

global optimum for all UI systems considered in this section. The second class, labelled $d(s) \to 0$, sets the benefit duration to the lowest possible value of one period, practically converting UI into a severance-pay-like system with a one-time payment in the first period after job loss. This class represents a local optimum, as seen in figure 3. \textsuperscript{45}

If we allow for differentiation of benefits over the business cycle, it is clear from table 2 that replacement ratios should be substantially higher during recessions. This is in line with the findings in the theoretical literature, including Landais et al. (2010) and Kroft and Notowidigdo (2011). In both UI classes shown here, payments that occur during recessions are more than twice as high as those in the good state.

A striking observation that can be made in table 2 is how little the efficient response of UI generosity to the state of the economy actually matters for welfare. While it is possible to improve welfare significantly over the status quo in the steady state – by almost 0.8% of the baseline welfare – the effect is much smaller in recessions.\textsuperscript{45}

\textsuperscript{45}While the Canadian EI system varies UI generosity by adjusting benefit durations, the policies shown in table 2 vary replacement ratios. As for the set of UI systems under considerations, which can be parameterized by $\rho(s)$ and $d(s)$ for $s \in \{0,1\}$, the optimal benefit duration is either one period or infinite, there is no optimal differentiation of benefit durations over the business cycle. Section 5 will provide more information on the effects of differentiated benefit durations.
Table 2: Welfare-maximizing UI Parameters

<table>
<thead>
<tr>
<th></th>
<th>Canadian EI</th>
<th>( d(s) = \infty )</th>
<th>( d(s) \to 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>differentiated</td>
<td>undifferentiated</td>
<td>differentiated</td>
</tr>
<tr>
<td>( \rho(0) )</td>
<td>55%</td>
<td>5.1%</td>
<td>4.7%</td>
</tr>
<tr>
<td>( \rho(1) )</td>
<td>55%</td>
<td>4.6%</td>
<td>4.7%</td>
</tr>
<tr>
<td>( d(0) ) (months)</td>
<td>10.1</td>
<td>( \infty )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>( d(1) ) (months)</td>
<td>9.2</td>
<td>( \infty )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>welfare change (total)</td>
<td></td>
<td>0.785%</td>
<td>0.784%</td>
</tr>
<tr>
<td>welfare change ( (s = 0) )</td>
<td></td>
<td>0.536%</td>
<td>0.501%</td>
</tr>
<tr>
<td>unemployment(^a)</td>
<td>9.87%/7.13%</td>
<td>7.22%/5.17%</td>
<td>7.20%/5.18%</td>
</tr>
<tr>
<td>market tightness(^a)</td>
<td>0.90/1.00</td>
<td>1.36/1.43</td>
<td>1.36/1.43</td>
</tr>
<tr>
<td>consumption</td>
<td>7.71</td>
<td>7.84</td>
<td>7.84</td>
</tr>
<tr>
<td>tax rate</td>
<td>3.73%</td>
<td>0.59%</td>
<td>0.59%</td>
</tr>
<tr>
<td>match quality</td>
<td>1.08</td>
<td>1.07</td>
<td>1.07</td>
</tr>
</tbody>
</table>

\(^a\) bad state/good state

Note: Optimal policies are based on a 4\(^{th}\)-order polynomial approximation of welfare as a function of \( \rho(s) \) calculated on a grid. The minimum spacing of this grid is 0.5%.

When choosing the best policy with infinite benefit duration – allowing for state-dependent benefits leads to improvements of no more than a hundredth of a percent compared to the best undifferentiated policy. Efficiency gains of this order of magnitude are almost certainly unnoticeable, and they are dwarfed by the effects of other choices regarding the design of the UI system.

When only the welfare changes in the bad state are considered, the cost of going with the best undifferentiated policy depends on how big the foregone differentiation actually is. Part of the reason for this result is the choice of a constant tax rate over time. Most of the additional costs of more generous benefits during recessions are paid for when when the economy is in the more prosperous state.

While these calculations qualitatively confirm the results of Andersen and Svarer (2010), Landais et al. (2010) and others that varying UI generosity over the cycle is desirable in
Figure 4: Optimal policy and the benefit of differentiation for alternative degrees of risk aversion

Note: The “match premium” is defined as difference between the average match quality and the average match quality of all job offers, which is unity.

principle, they also suggest that given the limited impact of this feature of the UI system, it may be better to look elsewhere for efficiency improvements. These findings are in line with the results obtained by Kristoffersen (2012), who reports that in a model with self-insurance calibrated to the US, the welfare improvements possible by adjusting UI generosity over the cycle are of the order of hundredths of a percent.

5 Robustness

The results presented so far regarding welfare-maximizing UI systems and the decomposition of welfare effects depend on a number of modelling and calibration choices. This section assesses the robustness of our findings under different assumptions regarding the degree of risk aversion and the role of self-insurance and discuss the sensitivity of the welfare measures to the labour market experience of very young households.

Figure 4 shows how the optimal replacement ratios in recessions depends on the replace-
ment ratio in normal times for risk aversions of $\sigma = 1, 5$. The results are presented in the same way as those for $\sigma = 3$ in figure 1 above. As before, the dark gray bands cover policy parameters that are within $\frac{1}{100}$% of the optimum in terms of welfare. The light gray areas cover the $\frac{1}{10}$% range. Benefit duration is fixed at 3 months for this exercise.

Clearly, optimal benefit generosity varies significantly with the households’ risk aversion. For log utility, the welfare-maximizing policy is $\rho(0) = 0$ for any level of $\rho(1)$. In the high risk aversion case $\sigma = 5$, optimal levels of $\rho(0)$ lie between 72% and 135%.

In both of these cases, there are levels of $\rho(0)$ for which choosing the optimal replacement ratio during recessions over an undifferentiated policy leads to welfare improvements of more than 0.1%. Yet, such gains are hardly a matter of fine tuning the UI system, as they are only possible if $\rho(1)$ is grossly suboptimal. In such a situation, it would be most effective to adjust $\rho(1)$.

In the $\sigma = 1$ case, the optimal differentiated and undifferentiated policies coincide at a corner result, $\rho(0) = \rho(1) = 0$. In the high risk aversion scenario, the optimal undifferentiated policy is $\rho(0) = \rho(1) = 93\%$. Switching to the best differentiated policy of $\rho(0) = 134\%$ and $\rho(1) = 91\%$ yields a welfare improvement of 0.037%.

While the optimal generosity of UI systems depends strongly on households preferences\textsuperscript{47}, the result that only small welfare improvements are possible by introducing business-cycle dependent benefits into an otherwise reasonably efficient system does not.

As much of the theoretical literature on UI and the business cycle employs models that do not allow for asset accumulation and self-insurance, it is interesting to consider such a scenario in the context of our model, too. Figure 5 shows optimal replacement ratios and benefit durations in a scenario where households are not allowed to save, so that UI benefits equal their consumption levels during unemployment. This scenario introduces welfare pay-
ments that guarantee a minimum income corresponding to 50% of the typical income of a 20-year-old household as well as pensions of 70% of the typical income of households before retirement. These benefits ensure that consumption is always positive.\textsuperscript{48}

Notice that without savings, the link between UI generosity in the good state and optimal benefits in the bad state almost disappears. In previous scenarios, the precautionary savings accumulated in the good state partly determined the need for insurance in the bad state; this is not the case in this scenario.

The welfare improvement resulting form replacing the constrained welfare-maximizing replacement ratio of $\rho(0) = \rho(1) = 85.4\%$ with the unconstrained optimum of $\rho(0) = 90.3\%$ and $\rho(1) = 84.6\%$ is 0.003\%. The effect of switching to a differentiated policy when varying benefit durations is even smaller.\textsuperscript{49} Generally, there is little scope for improving welfare by differentiating benefits in the neighbourhood of the constrained optimal policy. If the base policy is not chosen efficiently, however, adjusting UI generosity in the bad state can have a stronger positive effect on welfare, up to 0.64\% in this scenario. This welfare improvement

\textsuperscript{48}In this scenario, benefit duration is fixed at $d(0) = d(1) = 6$ when replacement ratios are varied and replacement ratios are set to $\rho(0) = \rho(1) = 60\%$ when optimal durations are analysed.

\textsuperscript{49}In this case, optimal differentiation is very small, with $d(0) = 5.97$ and $d(1) = 5.96$. 

24
can be even larger under a less generous welfare system.\footnote{In the lower left panel of figure 5 shows a plateau for replacement ratios below 30%. This is because the UI system becomes irrelevant if UI benefits are typically below welfare payments. The welfare benefits of 50% of young individuals' typical income, which were used in this simulation, are higher than what single households can actually receive in the form of social assistance and housing benefits. Cutting maximum welfare payments in half makes the welfare plateau smaller and increases the maximum possible welfare improvement from UI benefit differentiation to as much as 2.2% for values of $\rho(1) < 20\%$.}

Figure 6 demonstrates the importance of life-cycle effects for the evaluation of UI. The top left panel shows the welfare change by age that results from raising the replacement ratio from $\rho(0) = \rho(1) = 30\%$ to 40% for a fixed duration of 3 months. Agents up to age 35 tend to benefit from the change, while those older than that suffer a welfare reduction. The reason for this pattern is that borrowing constraints affect younger households, while those above age 40 are effectively unconstrained. This can be seen in the top right panel, which shows that consumption peaks at around age 40 and declines thereafter, which is to be expected for well-insured households given that the interest rate is below the discount rate. The bottom right panel shows how the liquidity constraints affect labour market outcomes. Unemployment rates rise with age up to 40 years, which reflects older households' choice for longer job search. This results in a better matches for older individuals - the average match quality is about 10% better at age 50 than at age 20.

As explained above, the welfare criterion used is utilitarian welfare of the part of the population that has had a job at some point in their lives. This is the whole simulated population excluding entrants into the labour market who have not yet found work. The reason is that the welfare of this particular group can have a substantial impact on the overall welfare measure, which is not robust to small changes in the calibration of the model.

Figure 7 shows the welfare impact of the same policy change as in figure 6, using an inclusive utilitarian measure as well as the welfare measure that excludes entrants. The top panels show welfare changes by age for different levels of initial endowments, which are measured in months of income.\footnote{As before, "income" refers to the income of an agent with productivity one on a job characterized by the average match quality of unity outside of a recession. This measure is very close to the average starting wage of individuals first entering the labour market.} The two welfare measures yield virtually identical

\begin{equation}
50
\end{equation}

\begin{equation}
51
\end{equation}
Figure 6: Age profiles

Note: The “match premium” is defined as difference between the average match quality and the average match quality of all job offers, which is unity.

results for household aged 21 and above. Only for 20 year old households, there is a stark difference. While those who are already employed enjoy a large welfare improvement due to higher benefits, those who are still unemployed suffer a strong negative effect. The reason is that the first group is comprised of liquidity constrained individuals who are entitled to insurance, whereas the second group has no entitlement but suffers from the reduced market tightness under more generous benefits. The two lower panels show the overall welfare effect of the policy change for different levels of endowment. While the welfare measure that excludes entrants yields rather predictable results – higher endowments mean less liquidity constraints and hence less appetite for UI – the inclusive welfare measure exhibits a non-monotonic relationship between benefit generosity and welfare.

The choice of the welfare measure does not affect the result that the welfare effects of business cycle dependent UI are small. An alternative experiment that increases the replacement ratio from 30% to 40% in recessions only rather than in both states as above produces almost exactly the same welfare patterns as those reported in figure 7, except that
all effects are a muted by a factor 10.

Table 3 shows the welfare-maximizing UI systems in the class under consideration, as in table 2 above, using the welfare measure that includes entrants. While the results are qualitatively similar to those shown in table 2, in that the optimal temporal differentiation of UI benefits is considerable while affecting welfare very little, including agents who are not yet entitled to receive benefits leads to substantially lower optimal values for the replacement ratios.

6 Conclusions

The main finding of this paper is that potential gains from allowing benefit generosity to be differentiated over the business cycle are small. They are small in absolute terms as long as the UI system is reasonably efficient otherwise. They are small relative to the efficiency improvements that can be achieved by other, more conventional means such as adjusting general replacement ratios and benefit durations for systems that are further away from the optimum.
Table 3: Welfare-maximizing UI Parameters

<table>
<thead>
<tr>
<th></th>
<th>Canadian EI</th>
<th>(d(s) = \infty)</th>
<th>(d(s) \to 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>differentiated</td>
<td>undifferentiated</td>
<td>differentiated</td>
</tr>
<tr>
<td>(\rho(0))</td>
<td>55%</td>
<td>5.6%</td>
<td>2.8%</td>
</tr>
<tr>
<td>(\rho(1))</td>
<td>55%</td>
<td>2.6%</td>
<td>2.8%</td>
</tr>
<tr>
<td>(d(0)) (months)</td>
<td>10.1</td>
<td>(\infty)</td>
<td>(\infty)</td>
</tr>
<tr>
<td>(d(1)) (months)</td>
<td>9.2</td>
<td>(\infty)</td>
<td>(\infty)</td>
</tr>
<tr>
<td>welfare change (total)</td>
<td>–</td>
<td>1.995%</td>
<td>1.985%</td>
</tr>
<tr>
<td>welfare change ((s = 0))</td>
<td>–</td>
<td>2.019%</td>
<td>1.899%</td>
</tr>
<tr>
<td>unemployment(^a)</td>
<td>9.87%/7.13%</td>
<td>7.23%/5.07%</td>
<td>7.08%/5.08%</td>
</tr>
<tr>
<td>market tightness(^a)</td>
<td>0.90/1.00</td>
<td>1.34/1.47</td>
<td>1.40/1.46</td>
</tr>
<tr>
<td>consumption</td>
<td>7.71</td>
<td>7.84</td>
<td>7.84</td>
</tr>
<tr>
<td>tax rate</td>
<td>3.73%</td>
<td>0.49%</td>
<td>0.48%</td>
</tr>
<tr>
<td>match quality</td>
<td>1.08</td>
<td>1.07</td>
<td>1.07</td>
</tr>
</tbody>
</table>

Note: Optimal policies are based on a 3\(^{rd}\)-order polynomial approximation of welfare as a function of \(\rho(s)\) calculated on a grid. The minimum spacing of this grid is 0.5%.

\(^a\) bad state/good state

On the one hand, this means that fine-tuning a UI system for optimal intertemporal patterns of generosity may not be worth it. On the other hand, even implementing inefficient business-cycle dependent policies is *per se* unlikely to do much damage.

As pointed out in the introduction, most of the existing literature on this particular feature of UI systems is based on models that do not allow for saving and self-insurance. This can lead to an exaggeration of the positive effects of UI, which may be quite strong even for low levels of risk aversion. Moreover, it may result in an emphasis of the fine tuning of subtile aspects of the system that can strongly affect individual behaviour when household consumption and thus indirectly behaviour is well controlled by policy; in more realistic settings, where the policy maker has substantially less control over households who can protect themselves effectively against the potential adverse effects of sophisticated incentive
schemes, the implementation details of a system may be less important.\footnote{An example is Werning (2002), who shows that the result that benefits should decline with the duration of an unemployment spell are not robust to allowing for hidden savings.}

Another aspect of model design that can be relevant when quantifying the welfare effects of UI is the choice between life-cycle and infinite-horizon models. It is important to note that in an infinite-horizon setting even with carefully calibrated, realistic income processes, there is no reason to expect that the resulting stationary wealth distribution bears much resemblance to the one that results under life-cycle behaviour. While it is possible to include features that help obtain a more realistic distribution\footnote{See Krusell and Smith (1998) for a classic example.}, simulating an actual life-cycle model appears to be the most straightforward and robust way to account for the different behavioural and welfare effects that policy has on different groups of the population.\footnote{While it is straightforward to model realistic life-cycle income profiles with all their implications for consumption and saving behaviour in life-cycle models, this is more of a challenge in infinite-horizon settings.} We have seen in section 5 that policy changes affect different age groups very differently. Modelling these groups explicitly therefore seems important for quantitative exercises.

As discussed above, young individuals who have not successfully entered the labour market yet are excluded from the welfare measure. This is done for reasons of robustness, and it seems justifiable in our context. However, when it comes to making policy recommendations regarding UI, this particular group is of great importance for overall welfare outcomes. This is because by definition, the share of these households that have to find a job is 100\%, which makes them extremely sensitive to changes labour market conditions. Moreover, under many UI systems, these individuals are not eligible to receive benefits, which means they only feel the effects of UI changes indirectly. Interestingly, the welfare of this groups depends on the very general equilibrium externalities that have been discussed in the context of benefit extensions and business-cycle dependent UI policies; see for example Landais et al (2010) and Lalive et al (2013). The problem of realistically calibrating the endowment of this group\footnote{This includes initial wealth, but also access to insurance from family and other welfare programs.} does not arise in infinite-horizon settings, and to my knowledge it has received little attention in the theoretical literature so far.
Appendix

A The Canadian EI System

For the purpose of administering EI, Canada is divided into 58 “EI Economic Regions,” which are chosen to cover individual labour markets or areas with homogenous labour market conditions. The geographical definition of these regions has been changed in the past. Figure 8 shows their current configuration, which has been in place since July 2000.

Figure 8: Employment Insurance Economic Regions


For each of these regions, the 3-months moving average of the seasonally adjusted unemployment rate is used to determine the conditions for the receipt of EI benefits. EI economic
Table 4: Regional Unemployment and Maximum Benefit Duration

<table>
<thead>
<tr>
<th>hours(^a)</th>
<th>unemployment rate less or equal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6%</td>
</tr>
<tr>
<td>420-454</td>
<td>0</td>
</tr>
<tr>
<td>455-489</td>
<td>0</td>
</tr>
<tr>
<td>490-524</td>
<td>0</td>
</tr>
<tr>
<td>525-559</td>
<td>0</td>
</tr>
<tr>
<td>560-594</td>
<td>0</td>
</tr>
<tr>
<td>595-629</td>
<td>0</td>
</tr>
<tr>
<td>630-664</td>
<td>0</td>
</tr>
<tr>
<td>665-699</td>
<td>0</td>
</tr>
<tr>
<td>700-734</td>
<td>14</td>
</tr>
<tr>
<td>735-769</td>
<td>14</td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
<tr>
<td>1715-1749</td>
<td>33</td>
</tr>
<tr>
<td>1750-1784</td>
<td>34</td>
</tr>
<tr>
<td>1785-1819</td>
<td>35</td>
</tr>
<tr>
<td>1820-</td>
<td>36</td>
</tr>
</tbody>
</table>

\(^a\) number of insurable hours of work during the qualifying period, which is typically the 52 weeks before job loss.

Regions are clustered into one of 12 groups based on this unemployment figure (no more than 6%, 6% to 7%, 7% to 8%, etc. up to 16% and more).\(^56\) The conditions for the receipt of benefit depend on the unemployment rate in a person’s region of residence. The higher this rate, the easier it is to qualify for benefits and the longer is the benefit duration. The required number of hours of work to qualify for benefits ranges from 420 in the 12 months prior to job loss (in regions with an unemployment rate above 13%) to 700 hours (if the

\(^56\) The official unemployment rates reported and used for this purpose by Human Resources and Skill Development Canada (HRSDC), the federal agency administering the unemployment insurance program, differ somewhat from those published by Statistics Canada, even though they are derived from the same survey. The most important difference is that in contrast to Statistics Canada, HRSDC includes aboriginal people living on reserves in their calculations. This leads to higher unemployment measures mostly in the relatively sparsely populated northern parts of the provinces.
The unemployment rate is 6% or less. The benefit duration increases with the number of hours worked up to a maximum between 36 weeks in low-unemployment regions and 45 weeks in high-unemployment regions. Table 4 shows the relationship between the regional unemployment rate and maximum benefit duration for a subset of the relevant qualification period brackets.\(^{57}\)

The aspect of the EI system that is most interesting for our purpose is how maximum benefit durations depend on regional unemployment rates. As can be seen in the last row of table 4, according to this criterion the EI economic regions can be classified into six groups.

Figure 9: Unemployment Rates across Economic Regions


\(^{57}\)Even though the requirements are stated in terms of insurable hours, the underlying idea is to model the system around a 35-hour workweek. The somewhat complicated pattern is as follows: For the same number of qualifying hours, the maximum benefit duration increases by 2 weeks per extra percentage point of regional unemployment. For every additional 70 hours (35 hours) of insurable work, the benefit duration increases by one week if the total number of hours is between 420 and 1,399 (between 1,400 and 1,820). The minimum number of hours required is 420 in regions with more than 13% unemployment. This minimum requirement increases by 35 hours for each one-percentage-point drop of local unemployment. No benefit durations of more than 45 weeks are possible.
Figure 9 reports the official unemployment rates in these regions for 2011-12.\textsuperscript{58} Clearly, the wide range of regional unemployment rates over which the generosity of the system varies has been chosen for a reason. Generally, unemployment rates tend to be higher in the four Atlantic Provinces Newfoundland and Labrador, Prince Edward Island, Nova Scotia and New Brunswick and relatively low in the Prairie Provinces Alberta, Saskatchewan and Manitoba. Within provinces, unemployment rates are typically higher in the northern EI regions.

EI is financed through contributions, which are adjusted annually. In 2013, the total contribution rate was 4.51% on earnings up to the maximum insurable amount of $47,400, which is also adjusted every year.\textsuperscript{59} This contribution rate is not an ideal indicator of the cost of UI in Canada, as the EI program does not only provide UI benefits, called regular benefits, but also parental benefits, sickness benefits and benefits to self-employed fishermen, among others. The ratios of regular benefits to insurable earnings and total employment income were 2.70% and 1.71%, respectively, in 2010.\textsuperscript{60}

\section*{B Business Cycle Dependent UI and Welfare}

Most of the simulation exercises presented in this paper address the question of how UI benefit differentiation over the business cycle affects welfare. This appendix presents a simple back-of-the-envelope calculation that suggests that – as long as the undifferentiated UI system is chosen optimally – switching to the optimal differentiated system is likely to have a very small impact on utilitarian welfare, independent of many of the specific assumptions made in this paper.

Consider an economy that can be in two states, $i = \ell, h$, which occur with probabilities

\textsuperscript{58}The official unemployment rate in regions 56 to 58, Yukon, the Northwest Territories and Nunavut, is 25%. Even though Statistics Canada measures the unemployment rates in these territories, the corresponding component of the Labour Force Survey is deemed experimental.

\textsuperscript{59}The contributions are split between employers and employees. Employers always pay 1.4 times the employees’ contributions. Since 2006, Quebec has had a different contribution rate, which is somewhat lower than the federal rate.

\textsuperscript{60}See EI Monitoring and Assessment Report 2012, Annexes 2.18 and 2.19.
\[ \pi^i. \] When in state \( \ell \), the economy is in a recession. Individuals can be either employed or unemployed. While employed, they enjoy a consumption level of \( c_e \) and when unemployed, their consumption level is \( c_u^i \), which may depend on the state \( i \) if the UI system is differentiated. Let \( U^i \) be the unemployment rate in each state and \( \bar{U} \) the average unemployment rate. \( u(c) \) is the individuals’ utility as a function of consumption.

Assume that the UI system influences households’ consumption. It is easy to derive an optimality criteria in the spirit of Baily (1978) for the consumption level of unemployed households under utilitarian welfare. For a differentiated system that controls \( c_u^\ell \) and \( c_u^h \) separately, the criteria are

\[
\frac{u'(c_u^i)}{u'(c_e)} = 1 + \epsilon^i, \tag{B.1}
\]

where \( \epsilon^i \) is the elasticity of unemployment duration with respect to \( c_u^i \) in each of the state. For an undifferentiated system that results in \( c_u = c_u^\ell = c_u^h \), the criterion becomes

\[
\frac{u'(c_u)}{u'(c_e)} = 1 + \bar{\epsilon}. \tag{B.2}
\]

\( \bar{\epsilon} = (\pi^\ell U^\ell \epsilon^\ell + \pi^h U^h \epsilon^h) / \bar{U} \) is the weighted average of the elasticities in the two macro states.

Calibrate the probabilities of the states as \( U^\ell = 9\% \), \( U^h = 7\% \) and \( \pi^\ell = 10\% \) and assume a constant relative risk aversion of 3. Under an undifferentiated system, let \( c^h = 1 \) and \( c^\ell = 0.9 \). This is in line with the findings of Gruber (1997), who reports that individuals losing their job experience an initial consumption drop of 6.8\%. Now suppose that the underlying UI system is optimal, so that condition (B.1) is satisfied. This implies that \( \bar{\epsilon} = 37.2\% \). Suppose further that in the bad state, there is no moral hazard problem, i.e. \( \epsilon^\ell = 0 \). Then, \( \epsilon^h \) must be 42.5\%.

What is the effect of switching to the optimal differentiated system? Under this system, individuals must be fully insured in the bad state. It turns out that \( c_e = c_u^\ell = 0.9998 \), \( c_u^h \) slightly drops to 0.8884, and \( U^h \) falls to 6.962\%. While there is a noticeable differentiation of the consumption levels of unemployed individuals over the business cycle under the optimal
policy\textsuperscript{61}, the resulting welfare improvement of 0.025\% is extremely small.

There are two main reasons for this result. Firstly, we implicitly allowed households to self-insure, which led to a moderate consumption drop during unemployment. If one insists that a replacement ratio of 50\% is associated with a 50\% consumption reduction after job loss, it is possible to find much larger welfare effects. Secondly, we assumed that the generosity of the undifferentiated system was chosen optimally. If this assumption is dropped, it is possible to obtain almost arbitrarily large welfare effects by switching from a very inefficient system to one which at least determines the benefit level in recessions efficiently.

\textbf{C Calibration}

All data on the Canadian economy used in the calibration comes from Statistics Canada.

The real interest rate was calculated as the average 10-year yield of Government of Canada marketable bonds from 2000:7 to 2012:12 minus CPI inflation\textsuperscript{62}.

Recession quarters since 1976 were determined using chain-weighted seasonally adjusted GDP\textsuperscript{63} data as periods with at least two consecutive drops in GDP. This procedure identified four recessions\textsuperscript{64} accounting for 15 out of the 150 quarters under consideration.

The average increase in the national unemployment rate during months that fall into recession quarters was 0.25\%. In all scenarios under consideration, $\lambda(0)$ was chosen to match this value. The matching rate $\omega(1)$ was calibrated to make the model match the average unemployment rate in non-recession months between 2000:7 and 2012:12, either at the national level or for groups of regions.

The productivity drop during recessions was calculated based on annual multifactor pro-

\begin{small}
\textsuperscript{61}The 11\% differentiation of consumption levels would likely be associated with a much larger differentiation of actual UI benefits if households self-insure.
\textsuperscript{62}Series v122487 and v41690914
\textsuperscript{63}Series v1992067 and v62305752
\end{small}
ductivity (MFP) estimates\textsuperscript{65} using the following regression:

$$\ln MFP_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 RQ_t,$$

where $RQ_t \in \{0, 0.25, 0.5, 0.75, 1\}$ is the share of recession quarters in year $t$. The value $\exp(\beta_3) = 0.949$ is the relative productivity during recessions.

Statistics Canada does not publish employment-related data by EI economic region,\textsuperscript{66} so some time series had to be constructed directly from the Labour Force Survey (LFS) microdata set.\textsuperscript{67} Labour force size, unemployment rate and unemployment duration series were constructed by region and month for EI regions 1 to 55 and 2000:7 to 2012:12. All series were seasonally adjusted. The labour force size was used for aggregating the regions into six groups and to compute national averages. The unemployment rates and durations by group of regions were used to calibrate the regional models. The national average of unemployment duration was also used in the calibration of the model to the aggregate Canadian economy.

\section*{D Model Solution and Simulation}

The household’s problem is solved by numerical dynamic programming, starting at the final period (number 1,440) and proceeding backward to the first. Value functions are represented on a multi-dimensional grid, using linear interpolation.\textsuperscript{68}

These grids are given by $AG \times MG \times SG$ for employed agents, $AG \times BG \times UG \times SG$ for unemployed agents and just $AG$ for retired agents. The possible macro states are represented by $SG \in \{0, 1\}$. $AG$ is the asset grid for values of $a_{it} \geq 0$. It is composed of a linearly spaced

\textsuperscript{65}Series v41712881
\textsuperscript{66}There are data by “economic region,” but the definition of regions used by Statistics Canada does not match EI economic regions.
\textsuperscript{67}The public-use file of the LFS does not contain geographic information that is more detailed than provinces. Access to the full dataset I used is restricted.
\textsuperscript{68}An exception to this rule are intervals in which the value function reaches its infimum, $-\infty$ for our parametrization. In this case, a non-linear function with the proper asymptotic behaviour is used. Even though linear interpolation may appear inferior to more sophisticated interpolation techniques such as higher order splines, its simplicity makes it so much faster that it becomes possible to use much finer grids in the computation, resulting improved accuracy.
section close to zero and a geometrically spaced section for higher asset levels. It has up to 497 nodes, although the grids for younger agents omit some of the higher nodes that cannot be reached to save space and computation time. $MG$ represents levels of match quality. It has 15 nodes at the midpoints of their respective quantiles plus three extra nodes at 50% and 75% of the lowest and 150% of the highest quantile nodes. The benefit grid $BG$ is a 19-point grid derived from $MG$, but adjusted to cover the whole relevant range of wages across age groups. Finally, $UG$ represents the remaining benefit duration as well as special states such as being in the waiting period. Its size depends on the maximum benefit duration and the complexity of the rules, but it is typically 7 nodes or less.

To solve the optimization problem of unemployed households, it is necessary to calculate the expectation over the continuous match quality variable $m$ as shown in equation 2. This is done using a quadrature-like approach, based on the grid points $MG$ chosen along this dimension and using the appropriate probability weights.

Solving the general equilibrium problem involves finding the levels of market tightness $\theta(0), \theta(1)$ that yield zero expected profits for jobs created in the bad and the good state, respectively. It also requires finding the tax rate that balances the government’s budget. To solve for the market tightness, I start from a reasonable guess, solve the household’s problem for the implied job offer rates $\omega(s)$, and then simulate the economy for a large number of periods to calculate aggregate variables, including the firms’ profits. If the zero-profit condition is not met with the required accuracy, I update my guesses of $\theta(s)$ and start over. In most scenarios, it takes about 5-10 of these iterations to determine market tightness.

All these computations are done for a tax rate of zero. Once the equilibrium market tightness has been determined, it is straightforward to calculate the tax rate required to balance the government budget based on measured expenditures and the tax base. All that needs to be done at this point is to adjust some of the results, including consumption and welfare measures, for the tax rate. This can be done so easily because the preferences are CRRA and the tax base includes all sources of household income except interest.
Solving the household’s problem for a typical set of parameters takes about 1-2 minutes (on one processor core) and requires approximately 2GB of memory. Given the large extent of the history simulated based on the household’s behaviour (see footnote 36), this part of the computation takes much longer, approximately 10-20 minutes. It thus takes about 1-2 hours to solve the full general equilibrium model for one set of policy parameters.\(^{69}\)

The finite size of the simulated sample as well the deviation permitted by the convergence criterion lead to noise in the variables obtained in the simulation runs.\(^{70}\) Table 5 summarizes the implications for some variables of interest.

<table>
<thead>
<tr>
<th>variable</th>
<th>coefficient of variation</th>
<th>max. rel. deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>welfare(^{a,b})</td>
<td>1.02 (\cdot) 10(^{-5})</td>
<td>2.78 (\cdot) 10(^{-5})</td>
</tr>
<tr>
<td>expected lifetime utility(^b)</td>
<td>5.59 (\cdot) 10(^{-5})</td>
<td>4.13 (\cdot) 10(^{-4})</td>
</tr>
<tr>
<td>(\theta(0))</td>
<td>3.39 (\cdot) 10(^{-4})</td>
<td>1.16 (\cdot) 10(^{-3})</td>
</tr>
<tr>
<td>(\theta(1))</td>
<td>9.32 (\cdot) 10(^{-5})</td>
<td>2.86 (\cdot) 10(^{-4})</td>
</tr>
<tr>
<td>unemployment rate ((s = 0))</td>
<td>1.85 (\cdot) 10(^{-4})</td>
<td>4.46 (\cdot) 10(^{-4})</td>
</tr>
<tr>
<td>unemployment rate ((s = 1))</td>
<td>5.56 (\cdot) 10(^{-5})</td>
<td>1.55 (\cdot) 10(^{-4})</td>
</tr>
<tr>
<td>consumption</td>
<td>6.60 (\cdot) 10(^{-6})</td>
<td>2.39 (\cdot) 10(^{-5})</td>
</tr>
<tr>
<td>tax rate</td>
<td>5.65 (\cdot) 10(^{-5})</td>
<td>2.01 (\cdot) 10(^{-4})</td>
</tr>
</tbody>
</table>

\(^{a}\) utilitarian welfare excluding individuals who have never worked

\(^{b}\) measured in consumption units

Note: Coefficient of variation and maximum relative deviation from the mean found in 200 simulation runs using different starting values.

References


\(^{69}\) Combined, all calculations performed for this version of the paper took about 4\(\frac{1}{2}\) core-years to complete.

\(^{70}\) To further improve the accuracy of the simulation, all results are averaged over ten simulation runs with different simulated samples.


